CHAPTER 12

LINEAR PROGRAMMING

POINTS TO REMEMBER

- Linear programming is the process used to obtain minimum or maximum value of the linear objectives function under known linear constraints.
- **Objective Functions:** Linear function z = ax + by where a and b are constants, which has to be maximized or minimized is called a linear objective function.
- **Constraints:** the linear inequalities or inequations or restrictions on the variables of a linear programming problem.
- **Feasible Region:** It is defined as a set of points which satisfy all the constraints.
- **To Find Feasible Region:** Draw the graph of all the linear in equations and shade common region determined by all the constraints.
- **Feasible Solutions:** Points within and on the boundary of the feasible region represents feasible solutions of the constraints.
- **Optimal Feasible Solution:** Feasible solution which optimizes the objective function is called optimal feasible solution.

Long Answer Type Questions (6 Marks)

1. Solve the following L.P.P. graphically

   Minimise and maximise $z = 3x + 9y$

Subject to the constraints $x + 3y \leq 60$

$$x + y \geq 10$$

$$x \leq y$$

$$x \geq 0, y \geq 0$$

2. Determine graphically the minimum value of the objective function $z = -50x + 20y$, subject to he constraints.

$$2x - y \geq -5$$

$$3x + y \geq 3$$

$$2x - 3y \leq 12$$

$$x \geq 0, \quad y \geq 0$$

3. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants per day, while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to produce atleast 60 shirts and 32 pants at a minimum labour cost? Solve the problem graphically.

4. There are two types of fertilisers A and B. A consists of 10% nitrogen and 6% phosphoric acid and B consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If A costs Rs. 6 per kg and B costs Rs. 5 per kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at minimum cost. What is the minimum cost?

5. A man has Rs. 1500 to purchase two types of shares of two different companies S_1 and S_2. Market price of one share of S_1 is Rs. 180 and S_2 is Rs 120. He wishes to purchase a maximum of ten shares only. If

one share of type S_1gives a yield of Rs. 11 and of type S_2 yields Rs. 8 then how much shares of each type must be purchased to get maximum profit? And what will be the maximum profit?

6. A company manufactures two types of lamps say A and B. Both lamps go through a cutter and then a finisher. Lamp A requires 2 hours of the cutter's time and 1 hours of the finisher's time. Lamp B requires 1 hour of cutter's and 2 hours of finisher's time. The cutter has 100 hours and finisher has 80 hours of time available each month. Profit on one lamp A is Rs. 7.00 and on one lamp B is Rs. 13.00. Assuming that he can sell all that he produces, how many of each type of lamps should be manufactured to obtain maximum profit?

7. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for atmost 20 items. A fan and sewing machine cost Rs. 360 and Rs. 240 respectively. He can sell a fan at a profit of Rs. 22 and sewing machine at a profit of Rs. 18. Assuming that he can sell whatever he buys, how should he invest his money to maximise his profit?

8. If a young man rides his motorcycle at 25 km/h, he has to spend Rs. 2 per km on petrol. If he rides at a faster speed of 40 km/h, the petrol cost increase to Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to cover the maximum distance within one hour. Express this as L.P.P. and then solve it graphically.

9. A producer has 20 and 10 units of labour and capital respectively which he can use to produce two kinds of goods X and Y. To produce one unit of X, 2 units of capital and 1 unit of labour is required. To produce one unit of Y, 3 units of labour and 1 unit of capital is required. If X and

Y are priced at Rs. 80 and Rs. 100 per unit respectively, how should the producer use his resources to maximise the total revenue?

10. A factory owner purchases two types of machines A and B for his factory. The requirements and limitations for the machines are as follows:

Machine	Area Occupied	Labour Force	Daily Output(In units)
A	1000 m^2	12 men	50
B	1200 m^2	8 men	40

He has maximum area of 7600 m^2 available and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximise the daily output?

11. A manufacturer makes two types of cups A and B. Three machines are required to manufacture the cups and the time in minutes required by each in as given below:

Types of Cup	**Machines**		
	I	**II**	**III**
A	12	18	6
B	6	0	9

Each machine is available for a maximum period of 6 hours per day. If the profit on each cup A is 75 paisa and on B is 50 paisa, find how many cups of each type should be manufactures to maximise the profit per day.

12. A company produces two types of belts A and B. Profits on these belts are Rs. 2 and Rs. 1.50 per belt respectively. A belt of type A requires twice as much time as belt of type B. The company can produce at most 1000 belts of type B per day. Material for 800 belts per day is

available. At most 400 buckles for belts of type A and 700 for type B are available per day. How much belts of each type should the company produce so as to maximize the profit?

13. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each first class ticket and a profit of Rs. 300 is made on each second class ticket. The airline reserves at a least 20 seats for first class. However at least four times as many passengers prefer to travel by second class than by first class. Determine how many tickets of each type must be sold to maximize profit for the airline.

14. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods A and B are available at a cost of Rs. 5 and Rs. 4 per unit respectively. One unit of food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories whereas one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the food A and B should be used to have least cost but it must satisfy the requirements of the sick person.

15. Anil wants to invest at most Rs, 12000 in bonds A and B. According to the rules, he has to invest at least Rs. 2000 in Bond A and at least Rs. 4000 in bond B. If the rate of interest on bond A and B are 8% and 10% per annum respectively, how should he invest this money for maximum interest? Formulate the problem as L.P.P. and solve graphically.

LINEAR PROGRAMMING

One Mark Questions

Chose the Correct option for the following MCQ's(Single option is correct).

1. Objective Function of a L.P.P. is

   (a) A constraint

   (b) A function to be opptimised

   (c) A relation between the variables

   (d) None of these

2. The optimal value of the objective function is attained at the points:

   (a) Given by intersections of equations with axis only

   (b) Given by intersections of inequations with x-axis only

   (c) Given by corner points of the feasible region

   (d) None of these

3. The solution set of the inequation $2x + y > 5$ is

   (a) Open half-plane that contains the origin

   (b) Open half-plane not containing the origin

   (c) Whole xy-plane except the points lying on the line $2x + y = 5$

   (d) None of these

4. If the constraints in a liner programming problem are changed, then

   (a) The problem is to be re-evaluled

   (b) Solution not defined

   (c) The objective function has to be modified

   (d) The change in constraints is ignored

5. Which of the following statements is correct?

(a) Every L.P.P. admits an optimal solution

(b) A L.P.P. admits unique optimal solution

(c) If a L.P.P. adimits two optimal soluctions it has an infinite number of optimal solutions

(d) None of these

6. Solution set of inequation $x \geq 0$ is

(a) Half-plane on the left of y-axis.

(b) Half-plane on the right of y axis excluding the points on y-axis.

(c) Half-plane on the right of y-axis including the points on y-axis.

(d) None of these

7. Solution set of the inequation $y \leq 0$ is

(a) Half-plane below the x-axis **excluding** the points on x-axis

(b) Half-plane below the x-axis including the point on x-axis.

(c) Half-plane above the x-axis.

(d) None of these

8. Regions represented by equations $x \geq 0$, $y \geq 0$ is

(a) first quadrant (b) Second quadrant

(c) Third quadrant (d) Fourth quadrant

Answers

1. Min z = 60 at x = 5, y = 5

   Max z = 180 at the two corner points (0, 20) and (15, 5).

2. No minimum value

3. Minimum cost = Rs. 1350 at 5 days of A and 3 days of B.

4. 100 kg of fertiliser A and 80 kg of fertilisers B; minimum cost Rs. 1000.

5. Maximum Profit = Rs. 95 with 5 shares of each type.

6. Lamps of type A = 40, Lamps of type B = 20.

7. Fan: 8; Sewing machine: 12, Maximum Profit = Rs. 392.

8. At 25 km/h he should travel 50/3 km, at 40 km/h, 40/3 km. Maximum distance 30 km in 1 hr.

9. X: 2 units; Y: 6 units; Maximum revenue Rs. 760.

10. Type A: 4; Type B: 3

11. Cup A: 15; Cup B: 30

12. Maximum profit Rs. 1300, No. of belts of type A = 200 No. of belts of type B = 600.

13. No. of first class ticket = 40, No. of second class ticket = 160.

14. Food A: 5 units, Food B: 30 units

15. Maximum interest is Rs. 1160 at (2000, 10000)

LINEAR PROGRAMMING
ONE MARK QUESTIONS ANSWER

1. (d)

2. (c)

3. (b)

4. (a)

5. (c)

6. (c)

7. (b)

8. (a)

CHAPTER 13

PROBABILITY

POINTS TO REMEMBER

- **Conditional Probability:** If A and B are two events associated with any random experiment, then P(A/B) represents the probability of occurrence of event A knowing that event B has already occurred.

  $$P\ (A/B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

  P(B) ≠ 0, means that the event should not be impossible.

  $$P(A \cap B) = P(A \text{ and } B) = P(B) \times P(A/B)$$

- Similarly $P(A \cap B \cap C) = = P(A) \times P(B/A) \times P(C/AB)$

  P (A/S) = P(A), P(A/A)= 1, P(S/A)= 1, P(A^1/B)= 1-P (A/B)

- **Multiplication Theorem on Probability:** If the event A and B are associated with any random experiment and the occurrence of one depends on the other, then

  $$P(A \cap B) = P(A) \times P(B/A) \text{ where } P(A) \neq 0$$

- When the occurrence of one does not depend on the other then these event are said to be independent events.

  Here P(A/B) = P(A) and P(B/A) = P(B)

  $P(A \cap B) = P(A) \times P(B)$

- Theorem on total probability: If $E_1, E_2, E_3 \ldots, E_n$ be a partition of sample space and $E_1, E_2 \ldots, E_n$ all have non-zero probability. A be any event associated with sample space S, which occurs with $E_1, or\ E_2, \ldots, or\ E_n$, then

  $$P(A) = P(E_1) . P(A/E_1) + P(E_2) . P(A/E_2) + \ldots\ldots + P(E_n) . P(A/E_n)$$

If A & B are independent then (i) A ∩ B^c, (ii) $A^c \cap B$& (iii) $A^c \cap b^c$ are also independent.

- **Bayes' theorem** : Let S be the sample space and $E_1, E_2 \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with $E_1, or\ E_{2\ or}\ \ldots, E_n$, then

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=0}^{n} P(E_i)P(A/E_i)}$$

- **Random variable** : It is real valued function whose domain is the sample space of random experiment.

- **Probability distribution** :It is a system of number of random variable (X), such that

X:	X_1	X_2	$X_{3\ldots}$	X_n
P(X):	$P(X_{1)}$	$P(X_2)$	$P(X_{3)\ldots}$	$P(X_{n)}$

Where P(x_i)> 0 and$\sum_{i=0}^{n} P(x_i) = 1$

- Mean or expectation of a random variables (X) is donated by E(X)

$$E(X) = \mu = \sum_{i=0}^{n} x_{i\ P(x_i)}$$

- Variance of X denoted by var(X) or σ_{x^2} and

$$Var(X) = \sigma_{x^2} = \sum_{i=0}^{n} (x_i - \mu)^2\, P(X_i) = \sum_{i=0}^{n} x_i^2 P(x_i) - \mu^2$$

- The non-negative number $\sigma_x = \sqrt{var(X)}$ is called standard deviation of random variable X.

ONE MARK QUESTIONS

1. Find P (A/B) if P(A)= 0.4, P(B)= 0.8 and P (B/A)= 0.6

2. Find P ($A \cap B$) if A and B are two events such that P(A) = 0.5, P(B) = 0.6 and P ($A \cup B$) = 0.8

3. A soldier fires three bullets on enemy: The probability that the enemy will be killed by one bullet is 0.7. What is the probability that the enemy is still alive?

4. If P(A)= $\frac{1}{2}$, P(B)= $\frac{7}{12}$ and P (not A or not B)=$\frac{1}{4}$. State whether A and B are independent.

5. Three coins are tossed once. Find the probability of getting at least one head.

6. Find P(A/B), if P(B) = 0.5 and P($A \cap B$) = 0.32

7. An urn contains 6 red and 3 black balls. Two balls are randomly drawn. Let x presents the number of black balls. What are the possible value of x?

8. A die is tossed thrice. Find the probability of getting an even number at least once.

9. Events E and F are such that P (Not E or Not F) = 0.25. State whether E and F are mutually exclusive.

10. Out of 30 consecutive integers 2 are choosen at random. Find the probability so that their sum is odd.

11. If event A and B are mutually exclusive and exhaustive events and $P(A) = \frac{1}{3}P(B)$ then Find P(A).

12. A natural number x is choosen at random from the first hundred natural numbers. Find the probability such that $x + \frac{1}{x} < 2$

13. A long contains 50 tickets numbered 1, 2, 3, ... 50 of which five are drawn at random and arranged in ascending order of magnitude. ($x_1 < x_2 < x_3 < x_4 < x_5$). What is the probability that $x_3 = 30$

TWO MARK QUESTIONS

1. If A and B are two events such that $P(A) \neq 0$, then find P (B/A) if (i) A is a subset of B (ii) $A \cap B = \phi$

2. A random variable X has the following probability distribution find K.

X	0	1	2	3	4	5
P(X)	$\frac{1}{15}$	K	$\frac{15K-2}{15}$	K	$\frac{15K-1}{15}$	$\frac{1}{15}$

3. If $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$, and P(B) =q find the value of q if A and B are (i) Mutually exclusive (ii) independent events.

4. If $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then find P(B/A) + P(A/B)

5. A die is rolled if the out come is an even number. What is the probability that it is a prime?

6. If A and B are two-events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and P(A I B) = $\frac{1}{8}$ Find P (not A and not B).

7. The probability that atleast one of the two events A and B occurs is 0.6. If A and B occur **simultaneously** with probability 0.3, then evaluate $P(\bar{A}) + P(\bar{B})$.

8. Three events A, B and C have probabilities $\frac{2}{5}, \frac{1}{3}$, and $\frac{1}{2}$, respectively. If $P(A \cap C) = \frac{1}{5}$ and $P(B \cap C) = \frac{1}{4}$, then find the values of P(C/B) and $P(A' \cap C')$.

9. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Jet A be the event "number obtained is even" and. B be B the event "number obtained is red". Find if A and B are independent events.

10. An urn contain 10 black and 5 white balls. Two balls are drawn from the **urn** one after the other without replacement. What is the porbability that both drawn balls are black?

11. Prove that if E and F are independent events, then events E and F′ are also indiependent.

12. The probability distribution of a discrete random variable X is given below

x	2	3	4	5
P(x)	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

find the value of K.

13. If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$, and $P(A \cap B) = \frac{4}{13}$, then find $P\left(\frac{A'}{B}\right)$.

14. In a class XII of a school, 40% of studints study mathematics,30% of the students. Study Biology and 10% of the class study both Mathematics and Biology. If a student is selected at random form the class, then find the probability that he will be studying Mathematics or Biology.

15. If P(A) = 0.4, P(B) = 0.8 and $P\left(\frac{B}{A}\right) = 0.6$ **then** find $P(A \cup B)$.

16. A die has two faces each with number 1, three faces each with number 2 and one face with number 3. If die is **rolled** once, then determine probability of not getting 3.

17. A coin is tossed 4 times. Find the mean and variance of the probability distribution of the number of tails.

18. There are 25 tic**kets bearing numbers** from 1 to 25 one tic**k**et is drown at random. Find the probability that the number on it is a multiple of 5 or 6.

Answers

ONE MARK QUESTIONS

1. 0.3
2. $\frac{3}{10}$
3. $(0.3)^3$
4. No
5. $\frac{7}{8}$
6. $\frac{16}{25}$
7. 0, 1, and 2.
8. $\frac{7}{8}$
9. Not mutually exclusive
10. $\frac{15}{29}$
11. $\frac{1}{4}$
12. 0
13. $P(E) = \frac{{}^{29}C_2 \subset {}^{20}C_2}{{}^{50}C_5}$

TWO MARK QUESTIONS

1. (i) 1 (ii) 0
2. K = $\frac{4}{15}$
3. (i) $\frac{1}{10}$ (ii) $\frac{1}{5}$
4. $7/12$
5. $1/3$
6. $3/8$
7. 1.1
8. 3/10
9. A and B are not independent events (s) because $P(A \cap B) \neq P(A) \cdot P(B)$
10. 3/7
11. –
12. 32
13. 5/9
14. 0.6
15. 0.96
16. 5/6
17. Mean = 2 and variance = 1
18. $\frac{9}{25}$